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PAPR REDUCTION TECHNIQUES FOR OFDM SYSTEMS

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DEDICATION

Dedicated to God, my beloved grand parents, family and friends.

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ABSTRACT

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Orthogonal Frequency Division Multiplexing (OFDM) is a promising technique for high data rate communications. However, one major drawback of OFDM systems is the high Peak to Average Power Ratio (PAPR) of the transmitted signal. There are several methods to address this problem, such as the partial transmit sequences (PTS), selective mapping (SLM) and clipping. In PTS, phase factors are used to combine the sub-blocks obtained from the partitioning of the OFDM data block. Since exhaustive search over all phase combinations is performed, the complexity of PTS increases exponentially with the number of sub-blocks. The reduced complexity PTS (RC-PTS) technique performs a gradient descent search on the phases with very little performance degradation.

In this report, we study the performance of PTS and RC-PTS techniques. We investigate their behavior for 16-QAM and 256 subcarrier systems. We propose a new technique of combining clipping with PTS and RC-PTS, and describe results showing complexity versus performance trade-offs of the proposed technique.

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Chapter 1

INTRODUCTION

Multi-carrier modulation systems use multiple signals over different carrier frequencies for transmission. The individual signals are called sub-carriers and the technique is coined as Frequency Division Multiplexing (FDM). The demands for high bandwidth efficiency in such multi-carrier systems have resulted in the evolution of techniques such as Orthogonal Frequency Division Multiplexing (OFDM) and MC-CDMA (Multi-carrier Code Division Multiple Access) [1] . OFDM has other advantages too, such as resistance to RF interference and robustness to multi-path fading. Digital audio and video broadcasting and Asymmetric Digital Subscriber Line (ADSL) modems are the examples of systems that use OFDM technology. The new generation wireless local area networks systems (WLANs), such as IEEE 802.11a in US and Hiperlan/2 in Europe, are based on these technology too [2].

The multi-path propagation arises from the multiple scattering by buildings, trees and other obstacles in the neighborhood of the transmission system. This induces significant changes in the amplitude and phase of the received signal. The multi-path fading also has the effect of delay in the arrival times, called as the delay spread, which also degrades the signal with Inter Symbol Interference (ISI). The mobility of the receiver also brings in Doppler effect, due to which the spectrum broadens. In a conventional serial transmission system, the frequency spectrum of the transmitted data is allowed to occupy the entire bandwidth, and this can easily be corrupted by the bursty Rayleigh fading in the channel [3]. In such systems, higher transmission rate can be achieved by higher modulation schemes at the expense of increased channel bandwidth, by decreasing symbol interval and by degrading the performance. The multiplexed or the parallel system overcomes most of the concerns in serial data transmission, by spreading out the fading among all its sub-carriers. In the parallel transmission, the total signal frequency band is divided into many non-overlapping sub-channels. Each sub-channel is modulated by a separate symbol and then all the sub-channels are multiplexed to form a symbol. In OFDM, the spectra of the individual channels are overlapped but they maintain the orthogonality constraints, which helps in the easy separation of the sub-channels at the receiver.

The OFDM avoids ISI by using guard intervals. This avoids complex channel equalization simply because the channel transfer function becomes constant over the bandwidth of all sub-carriers [4]. On the disadvantages of OFDM systems, they suffer from high peak-to-average power ratio (PAPR). This creates problem in the efficient use of power amplifiers.

1.1 PEAK TO AVERAGE POWER RATIO (PAPR) PROBLEM

The peak to average ratio is used to quantify the excursions of the envelope of a signal [5]. The peak to average power of a signal x_t is defined as,

$$PAPR = \frac{max|x_t|^2}{E[|x_t|^2]} \tag{1.1}$$

where $max|x_t|^2$ is the maximum instantaneous power and $E|x_t|^2$ denotes the average power of the signal. Many methods have been proposed in the literature to reduce the PAPR (Peak to Average Power Ratio) of OFDM signals. These include Clipping [6], Selective Mapping [7], Partial transmit sequences [8] and pulse-shaping [2]. Among all these methods, the Partial transmit sequences (PTS) is a distortion-less and phase modulation method showing very good performance when compared with other methods.

1.2 PARTIAL TRANSMIT SEQUENCES (PTS) TECHNIQUE

In this method, the transmitted signal is made to have a low peak power value by optimally combining the sub-blocks of the signal. The signal to be transmitted is split into a number of parts M, (M=8 in this work) and various phase combinations are used to combine the signal. Combining the signal in this manner would separate the sub-carriers that overlap with each other and add up to the peak power. The different combinations of phase vectors are tried and the phase vector for which the PAPR value is low is finally transmitted. The phase vectors are transmitted along with the signal for obtaining the signal back at the receiver.

1.3 REDUCED COMPLEXITY PTS AND CLIPPING

The PTS shows promising results in reducing the PAPR values but suffers from the problem of complexity at the transmitter. The complexity is due to the laborious search for the phase combination that can produce the lowest PAPR value. The reduced complexity algorithm of PTS performs the same operation of the ordinary PTS technique, while the complexity reduction is obtained by optimally searching the phase vectors for reducing PAPR values. Not all the phases are searched for, but the selection of the phases is done by using the concept of clustering. This reduction in the complexity is obtained at the cost of performance, but the amount of complexity reduction is more than the performance degradation which we see in results.

Clipping is a different PAPR reducing technique in which the transmitted signal is clipped before the amplification stage. This gives a good PAPR at the expense of inducing non-linearity and degradation in performance. The clipping in an OFDM signal is done either before or after the interpolation is performed. Interpolation in general is defined as the method of constructing new data points within the set of known discrete data points. Clipping the signal before the interpolation causes peak re-growth as interpolation now has to be done before the analog to digital conversion [6] . So, interpolation has to be done before the clipping to avoid the re-growth problem but this causes out-of-band power.

1.4 THE PRESENT WORK

In this report, we study and implement the PTS and the reduced complexity technique. This study is done for the OFDM system of different number of subcarriers, 64, 128 and 256. The study is also extended to 16-QAM modulated system. The various scenarios for the number of phase factors and number of iterations are also explored. Finally, we propose and study a new technique by combining clipping and PTS techniques. This particular idea reduces the complexity of the system but this is done at the expense of performance. The effect on the OFDM system is studied for different clipping levels and the CCDF plots for all the work are generated.

1.5 ORGANIZATION OF THE REPORT

Chapter 2 provides the background on OFDM systems. In chapter 3, a detail description of the PTS technique and its reduced complexity version is presented. The idea of combining clipping with the PTS techniques is presented in Chapter 4. Chapter 5 concludes this report and discusses directions for future work.

Chapter 2

ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

2.1 PRINCIPLE OF OFDM

OFDM (Orthogonal Frequency Division Multiplexing) is a special case of FDM(Frequency Division Multiplexing) and is truly considered as the combination of modulation and multiplexing. Multiplexing generally refers to the method of sharing the spectrum or the bandwidth with independent data channels or signals produced from different users. The signal itself is first split into independent channels, each modulated by a different data and then multiplexed back to create an OFDM carrier. The sub-carriers here should be orthogonal to each other to allow simultaneous transmission on a lot of sub-carriers in a tight frequency space without causing interference to each other.

The bit stream, originally in the serial form, is converted into the parallel form. Each stream of data is considered now as a sub-carrier and the IFFT is done for each sub-carrier separately. The original stream of data is considered to be in the frequency domain and the resultant signals after converting them to time domain are now added up to form a single OFDM packet.

Fading is a very important factor that has to be considered while designing a system. The fading is generally caused due to the reception of the signals at the receiver by different paths. The time delays of these signals results in the phase shifts, and when added to the main signal (LOS signal if any) causes serious degradation of the signal. The maximum time delay that occurs is called the delay spread, and based on this value, the channel can be classified as a flat or a frequency-selective fading channel. When the delay spread is less than a symbol period, it is called a flat fading channel, and when the delay spread is more than a symbol, it is called as a frequency-selective fading channel. Many CDMA systems perform poorly in frequency-selective fading channels, but OFDM gives a good performance in this kind of channel due to the fact that the different orthogonal frequencies of the OFDM do not get affected similarly at the same time. This is an advantage of the OFDM system.

Another important feature of OFDM is the cyclic-prefix. The interference induced by the delay spread affects the beginning of the next symbol. So to reduce the effect at the front of the symbol, the successive OFDM symbols are moved away from the region of the delay spread. The blank space is now being filled up by the end part of the symbol. This helps to push the start of the signal away from the delay spread zone and hence correctly estimate the phase of the next signal. This small part of the signal, filled in between the signals is called the 'cyclic-prefix' and is a key way of reducing the ISI(Inter Symbol Interference) in the OFDM system. The length of the cyclix prefix is set at around 10 percent to 25 percent of the symbol time. In practice, the length of the cyclic-prefix can be kept longer than the length of the impulse response of the channel if known. In the present work, a 5-tap channel is considered to evaluate the BER performance and so the length of the cyclic prefix in this report is set at 6.

2.2 SYSTEM MODEL

The functional block diagram of an OFDM system mainly comprises of three parts: Transmitter, Channel and Receiver.

2.2.1 TRANSMITTER

The starting point in the implementation is the generation of the bit-stream b_i. The bit stream is then modulated using Gray code constellation for QPSK or 16-QAM to obtain the symbols, s_k. Two bits constitute a symbol in QPSK and four bits constitute a symbol in 16-QAM. The symbols are then divided into the number of sub-carriers, N_c, using a Serial-to-Parallel (S/P) converter. The orthogonality and the multiplexing part of the OFDM is obtained by the successive IFFT and multiplexing blocks as shown in Fig. 2.1. The OFDM packets generated after the D/A conversion [8] are in the form of

$$x(t) = \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, 0 \leqslant t \leqslant NT \tag{2.1}$$

where X_n is an individual symbol in the block of N symbols, $X_n, n = 0, 1, ...N-1$ in an OFDM symbol formed with each symbol modulated to a set of N subcarriers, $f_n, n = 0, 1, ...N-1$.

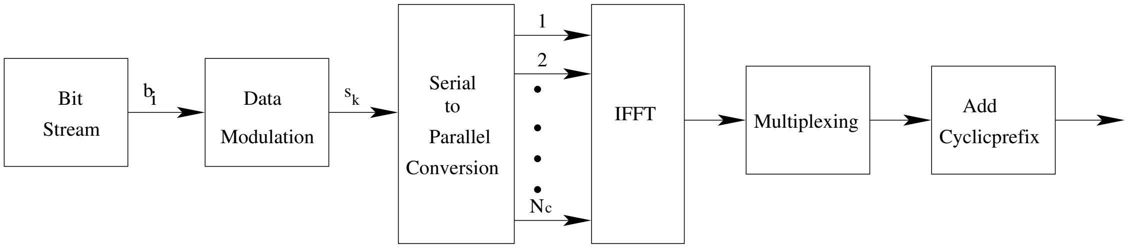

Figure 2.1: Transmitter block of the OFDM system.

The cyclic prefix is added just before the transmission of the packets into the channel. If the number of sub-carriers is N_c, and the length of the cyclic prefix or the guard interval is N_g, then the total length of the packet transmitted is simply $N_c + N_g$. The very important thing to be worked during the practical implementation at this stage is the normalization of the power for adding the cyclic prefix. Since the cyclic prefix is 6 in our work, the transmitting signal is normalized by $\sqrt{(N_c/N_c+6)}$. The pictorial representation of this is shown in Fig. 2.2.

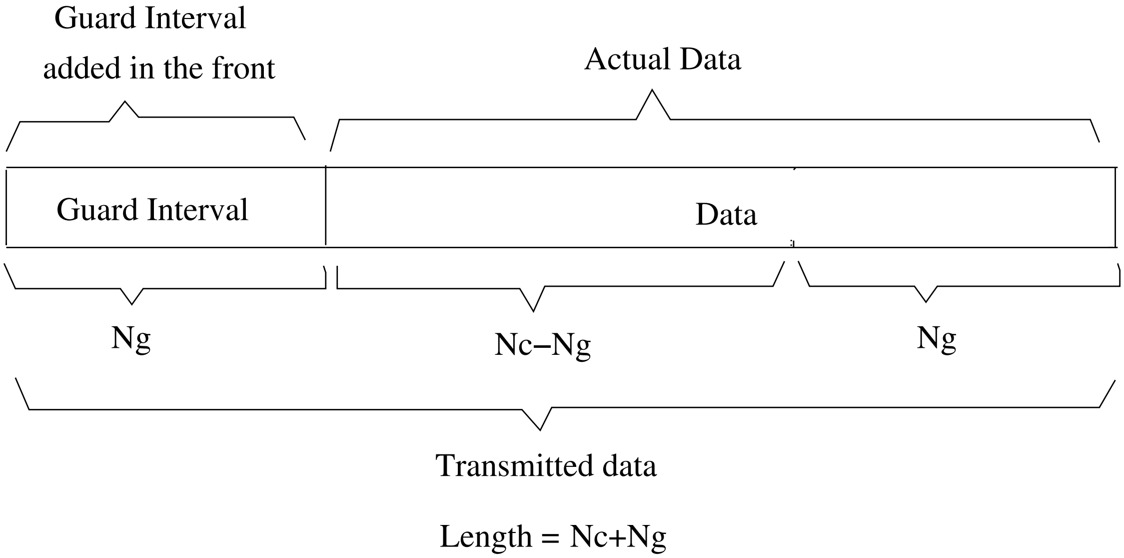

Figure 2.2: Cyclic prefix for the OFDM symbol.

2.2.2 CHANNEL

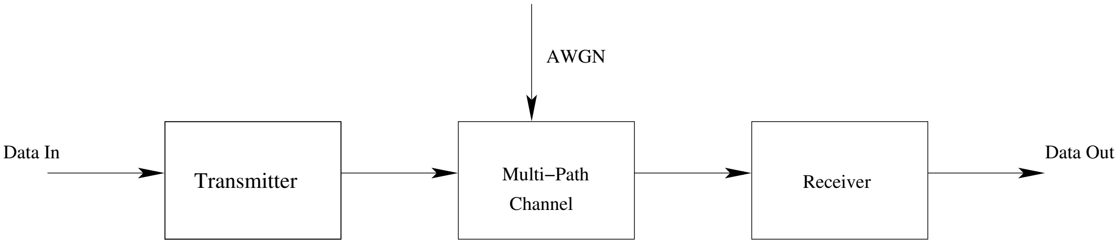

Figure 2.3: Block diagram of a Communication channel.

The channel part of the block involves the signal is being affected by the ISI and the Gaussian noise. The received signal vector $\mathbf{y}$ after propagation through the channel can be written as

$$\mathbf{y} = \widetilde{\mathbf{H}}\mathbf{x} + \mathbf{n} \tag{2.2}$$

where $\widetilde{\mathbf{H}}$ is the channel response matrix responsible for the ISI and the $\mathbf{n}$ represents the additive white Gaussian noise, of zero mean and variance, $\sigma_n^2 = N_o/2$.

2.2.3 RECEIVER

Almost all the stages at the receiver end can be viewed as the reverse processes of those applied at the transmitter end. The first thing done at the receiver is the removal of the cyclic prefix. The data is then converted back to parallel trains to make similar reverse operations as in transmitted using the S/P converter. The FFT operations are done on data pertaining to the sub-carriers and this data is then estimated for equalization. The Zero-Forcing (ZF) or the Minimum Mean Square Error (MMSE) equalizers can be used at this stage. In the present work, the MMSE equalizer is used due to its added advantage of considering the effect of noise onto the corruption of the signal. The signal after the FFT block is given as,

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \tilde{\mathbf{n}} \tag{2.3}$$

where $\mathbf{H}$ is the diagonal matrix, $\mathbf{s}$ is the symbol vector before IFFT at the transmitter and $\tilde{\mathbf{n}}$ is the complex AWGN vector in frequency domain. The equalization procedure is given as,

$$\mathbf{G} = (\mathbf{H}^*\mathbf{H} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_{N_c})^{-1}\mathbf{H}^* \tag{2.4}$$

where σ_s^2 is the received energy per symbol, $E_s^{'}$, σ_n^2 is the noise power and $\mathbf{I}_{N_c}$ is the identity matrix of size $N_c \times N_c$. In this case, $\mathbf{G}$ is a diagonal matrix.

The output of the equalizer is now considered to be ISI free. The next stage now is to de-modulate the symbols and map the points back to the bits transmitted.

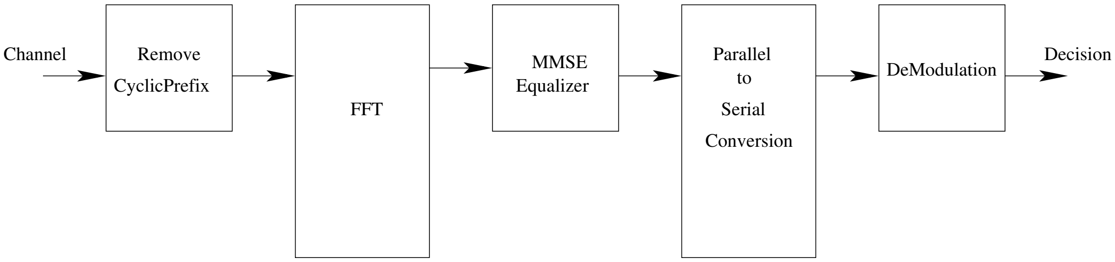

Figure 2.4: Receiver block of the OFDM system.

2.3 RESULTS

The error performance of the OFDM system using QPSK modulation is close to QPSK modulated packets in AWGN channel. In Fig. 2.5, we can observe that the error rate is as low as 10^{-5} for $10dB$ of Eb/No.

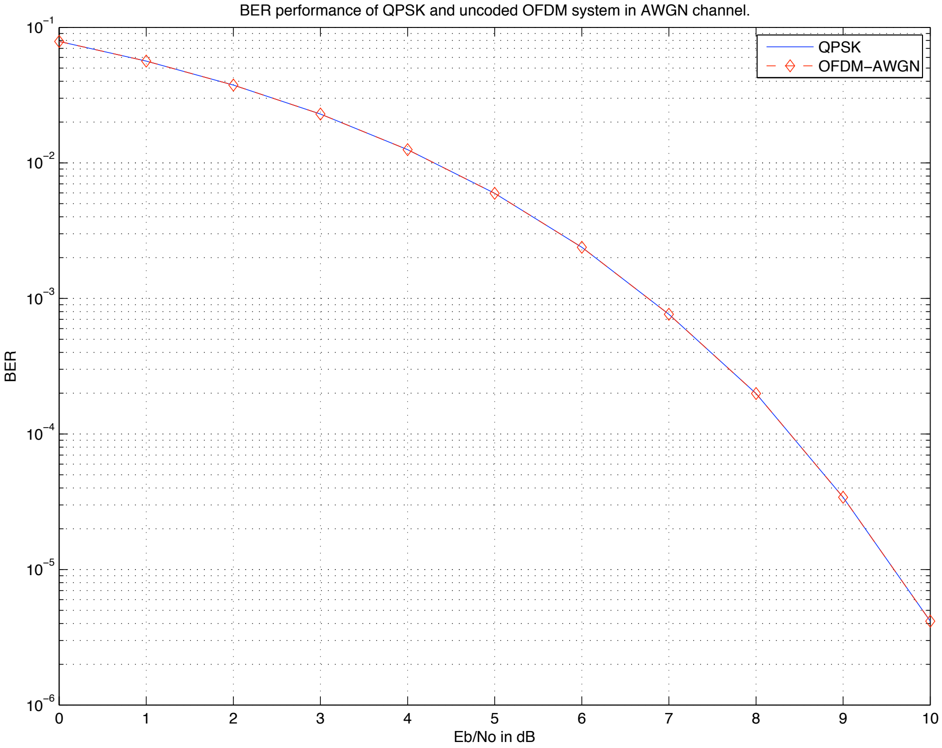

Figure 2.5: BER Performance of an uncoded OFDM system in an AWGN channel.

Fig. 2.6 shows the Gray code constellation mapping for QPSK and 16-QAM. The **I** and **Q** represent the in-phase and the quadrature component of the complex envelope of the symbol. In QPSK, two bits are mapped to one symbol and in 16-QAM, four bits are mapped to one symbol.

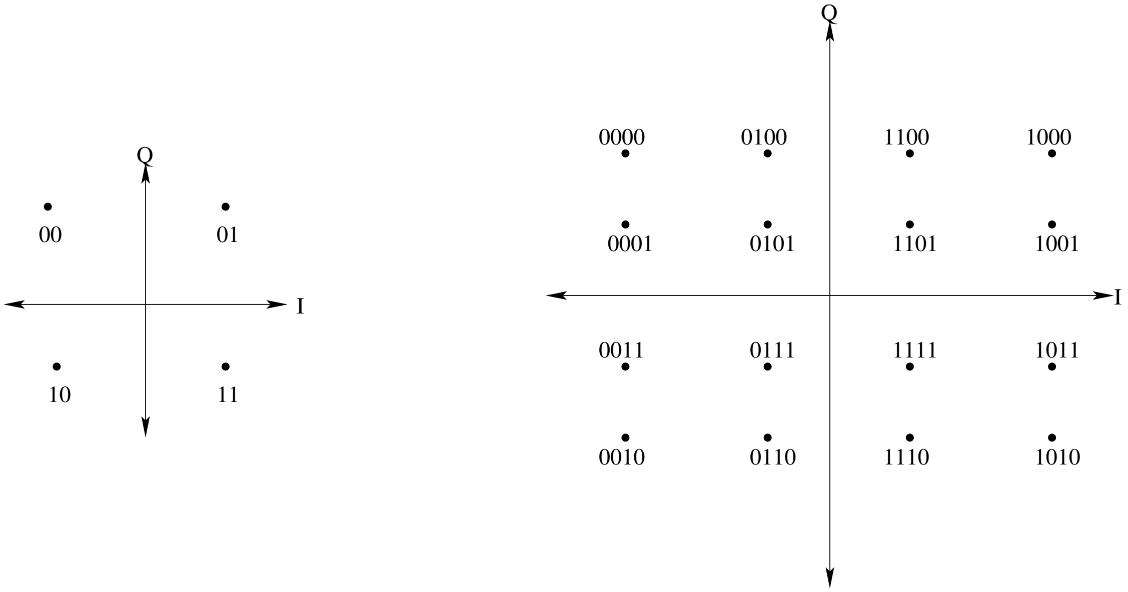

Figure 2.6: Constellation diagram for QPSK and 16-QAM mapping.

The OFDM can perform better than other modulation schemes in the presence of interference. The ideas of cyclic prefix, orthogonality, and multiplexing add up cumulatively to the performance of the system. The performance of the OFDM system in the static multi-path channel with an impulse response of $h =$[0.227 0.46 0.688 0.46 0.227]^T is shown in Fig. 2.7. The two curves depicts the performance of QPSK and 16-QAM modulated OFDM packets.

The channel used is a very bad channel having a spectral null in its frequency response. This can lead to symbols undergoing deep fading and can result in high BER. The QPSK results in Fig. 2.7 match perfectly with the results in [4]. The BER of the 16-QAM modulated OFDM packets is worse than that of the QPSK modulated packets for the same channel. This can be simply because the inter-

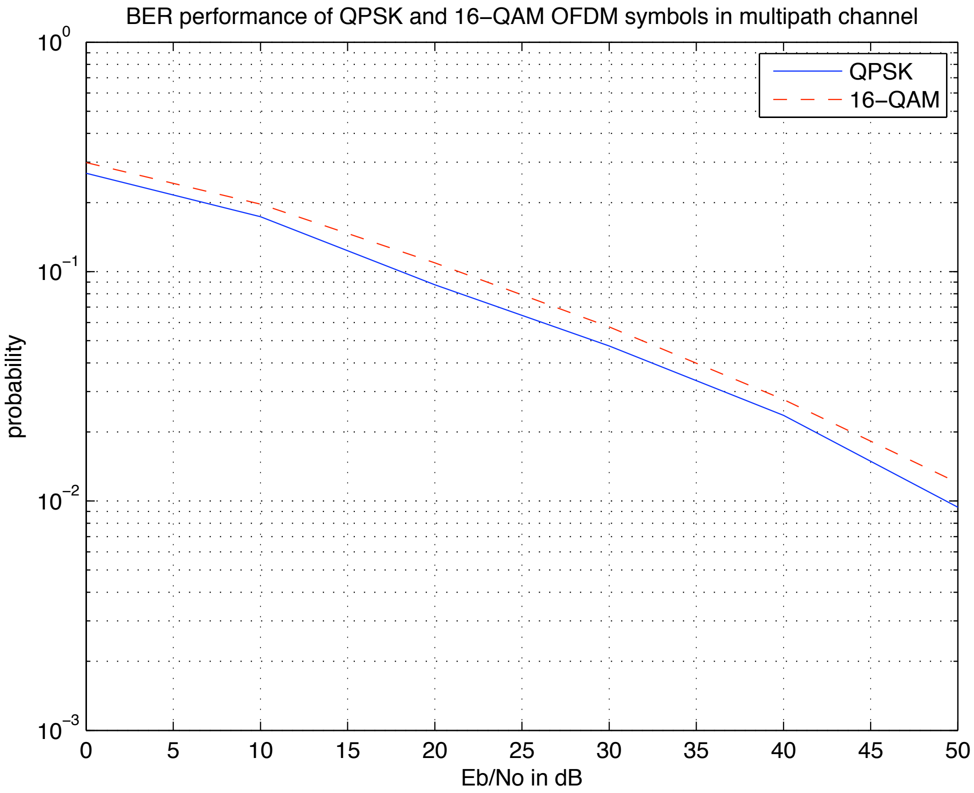

Figure 2.7: BER Performance of an uncoded OFDM system in a static multi-path channel.

constellation distance in 16-QAM is less when compared to QPSK and the effect of interference and noise would easily disturb the constellation points from their point.

Chapter 3

PTS AND REDUCED COMPLEXITY PTS (RC-PTS) TECHNIQUES

The multiplexing of different signals in OFDM produces overlapping on each other. For the case when all the tones share the same constellation, the maximum limit of the PAPR is given as

$$PAPR[x(n/L)] \leq N\frac{max|X_k|^2}{E[|X_k|^2]} \tag{3.1}$$

where $x(n/L)$ is the L times over-sampled discrete form of x_t and X_k is a QPSK or QAM symbol. The case of the equality occurs when all the QPSK or QAM modulated symbols, $k = 0, 1, 2, ...N - 1$ have the same phase and one of the constellation points with largest amplitude is selected [5]. The value of the PAPR grows with the number of tones and is generally proportional to the peak to average power ratio (PAPR) of the constellation. Since linear transmission of the OFDM signals requires linear operation over the range of $[-max|x_n|, max|x_n|]$, for zero distortion, the average power is at most $1/N$ times the maximum available power. So the base-band OFDM signal with N_c number of sub-carriers has a $PAPR = N_c^2/N_c = N_c$ and so for a 64 sub-carrier OFDM signal, $PAPR = 64 \approx 18.1dB$.

For adequate coverage and to minimize the battery consumption in wireless applications, efficient power amplification is very necessary [8]. OFDM signals thus having high PAPR values may require transmitters with linear output amplifiers with wide dynamic range that are both expensive and inefficient. Another solution to this problem can require significant back-off from the average operating point of the non-linear amplifier and this also causes significant power efficiency penalty. The power inefficiency leads to low battery power life for the mobile receiver and very high operating costs at the base-station. These factors can be considered to be the main reasons for the delay in the adaptation of OFDM systems into the WLAN products [9].

Several alternative procedures have been proposed to reduce the PAPR of the OFDM systems. Each procedure has advantages and disadvantages. Clipping was among the first techniques proposed to solve the problem and later techniques such as selective mapping, partial transmit sequences, dummy sequence insertions, ..etc. have evolved operating on specific features of the OFDM signals.

3.1 CLIPPING

The simplest technique known to reduce the PAPR problem in the OFDM system is to deliberately clip the output of the IFFT block at the transmitter side. The clipping of the signal can be done before or after the interpolation and each has its own problems. Clipping before the interpolation causes peak re-growth from analog to digital conversion and clipping after interpolation causes significant out-of-band power.

The performance of the clipping technique is good but this has to be done at the expense of performance degradation. The performance degradation is caused due to the removal of the important information in the signals. The additional non-linear distortion caused by clipping creates inter-modulation distortion that increases the BER of standard linear receivers and also causes the transmitted signal's spectral widening which in turn increases the adjacent channel interference to other users [5].

3.2 PARTIAL TRANSMIT SEQUENCES (PTS)

The selective mapping and the partial transmit sequences techniques have shown promising results in the PAPR reduction without any distortion being induced. Both the techniques can be considered as coding technologies where the transmitted signal is coded using phases and the additional information of the code, phase in these procedures is being transmitted along with the signal.

In the PTS approach, the input data block i.e., the output of the modulation scheme is partitioned into disjoint clusters called the sub-blocks. These clusters are optimally combined in a way to reduce the PAPR of the OFDM signal. Let the data block $X_n, n = 0, 1, 2, ...N-1$ be defined as a vector $X = [X_0 X_1 X_2 ......X_{N-1}]^T$. This vector is now partitioned into M disjoint sub-blocks $X_m, m = 1, 2, 3, ...M$ and these are represented as

$$X = \sum_{m=1}^{M} X_m \tag{3.2}$$

Over-sampling is done before this stage during the implementation. The symbol spaced sampling in the OFDM symbols sometimes misses some of the signal peaks, and can result in optimistic results for the peak power values [10]. For this reason, the signal samples are over-sampled by a factor of $L = 4$ in the implementation to estimate the true values of PAPR. The L times over-sampling the data is equivalent to padding $(N-1) * L$ zeros to the OFDM symbols.

The partitioning of the OFDM data block consists of contiguous set of subcarriers of equal size. The M partitions are now searched for the optimum combination of phases, $b_m = e^{\jmath\phi_m}, m = 1, 2, ...M$. The time-domain signal after combining the phases is expressed as x' and these sub-blocks of data are called as the partial transmit sequences. The data x' is given by

$$x' = \sum_{m=1}^{M} b_m x_m \tag{3.3}$$

The coefficients b_m are the weighted factors of rotations on which the working of the PTS technique is mainly focused on. The block diagram implementation of the technique is given in Fig. 3.1. The optimization block in the diagram searches for the optimum combination of the rotations for the M sub-blocks and the combination of phases producing the least PAPR value.

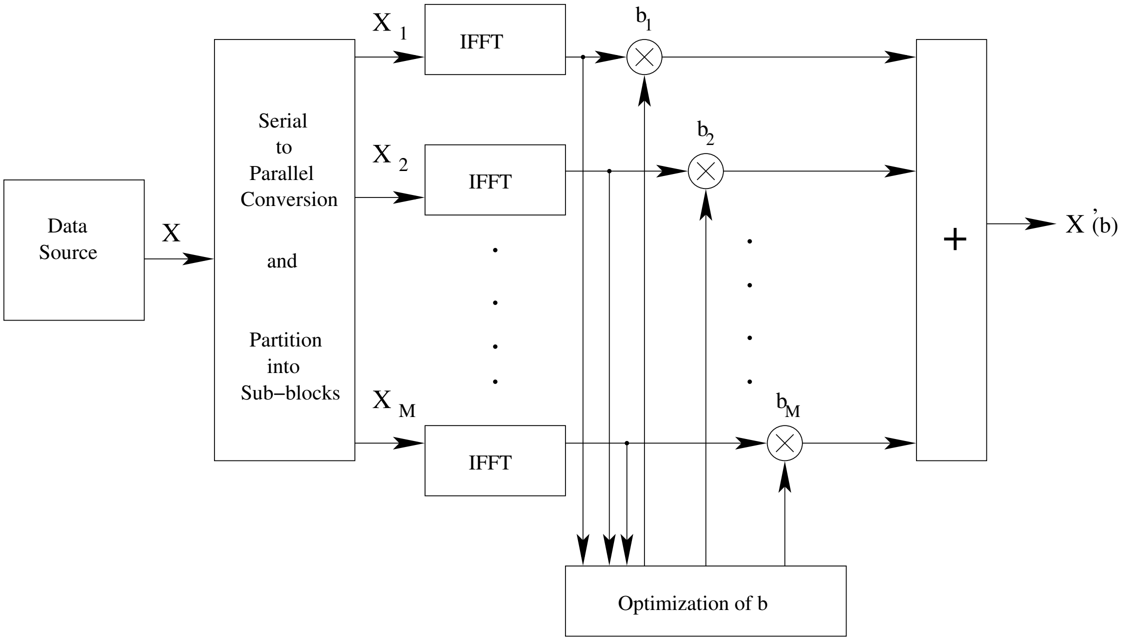

Figure 3.1: Partial Transmit Sequence approach for PAPR reduction in OFDM systems.

The selection of the phases is restricted over a specific set. This is done to set the limits on complexity produced by searching the entire set of possible phases. The set of allowed phase factors is given as

$$P = [e^{\jmath 2\pi l/W} | l = 0, 1, 2, ...W-1] \tag{3.4}$$

where W is the number of allowed phase factors considered. In this report, we have used $W = 4$. The four allowed phases thus would be $[+1, -1, +\jmath, -\jmath]$. Multiplying this phases as in Eq.(3.3) with the sub-carrier signals produces different ways in which they can be summed up to give an OFDM symbol with low peak power. This phases would now change the manner in which the sub-carriers are overlapped onto each other to produce different envelope powers to minimize the maximum value of PAPR equation of Eq.(1.1). Reducing the maximum power possible implies reducing the PAPR of the OFDM system.

The PTS technique has promising results in reducing the PAPR of the OFDM systems without inducing distortion and this is done at the cost of complexity. The complexity involved is the massive search for the phase factors that can achieve the lowest PAPR values. For M=8 and W=4, the number of phase combinations to be searched can be as high as, $4^8 = 65536$. The numerous number of IFFT blocks at the transmitter end also contribute highly to the complexity of the PTS technique. The overall search complexity increases exponentially with the number of partial transmit sequences M.

3.3 REDUCED-COMPLEXITY PTS TECHNIQUE (RC-PTS)

The reduced complexity algorithm proposed in [10] works on reducing the complexity of the original PTS technique by reducing the number of phase factors to be explored. Not all the phases are searched for to obtain the optimum combination of phases. The number of phases worked on are explored simultaneously in this technique by using the idea of Hamming distance. This step of not searching all the possible phases affects the performance of the system, but the tradeoff is best obtained between the complexity and performance. This means that for less sacrifice in performance, the gain in reducing complexity is high.

The RC-PTS starts from initially considering a vector of phase factors. The next step is to find an updated vector of phases in the neighborhood of the pre-determined vector that results in the largest reduction of PAPR. The neighborhood searched for is constrained to radius **r** that is defined as the set of vectors with Hamming distance equal to or less than **r** from its pre-determined vector. The equation used for the phase vector update is given as,

$$b' = arg[max_{||\hat{b}-b||_H \leq r}(PAPR(b) - PAPR(\hat{b}))] \tag{3.5}$$

where $||.||_H$ denotes the Hamming weight of the vector and **r** is the radius on which the neighborhood is defined. The number of vectors searched for in this algorithm is directly based upon this radius **r**, as this defines how close the search reaches the ordinary PTS. For the radius $\mathbf{r} = M$, the RC-PTS is same as the ordinary PTS technique as the search progresses in the vicinity of all the possible phases. The RC-PTS algorithm is summarized in the following way.

1. Partition the input data block into M sub-blocks.

2. Set the pre-determined vector, $\mathbf{b} = [11111111]^T$ and iteration count $\mathbf{i} = 1$.

3.Among the vectors in the vicinity of **b**, find $\mathbf{b}'$, achieving the largest difference in the PAPR.

4. Update **b** with $\mathbf{b}'$ if the PAPR attained is lower than that with the pre-determined vector and proceed to next step. Else terminate the loop.

5. If iteration count, **i** is less than the maximum count **I**, increase **i** by 1 and get back to step 3. Else terminate the loop.

The iteration count in the algorithm is used to get closer to the optimum phase combination from the vector for which the lowest PAPR is achieved in step 4. This idea is pretty close to the k-means clustering algorithm when the centroid of the cluster is updated repeatedly based on the metric of the Hamming distance.

The increase of the radius **r** from 1 to M pushes the performance curve towards the performance of the optimum ordinary PTS. The increase of **i** also helps to get close to the optimum curve as the phase search is intensified after updating the corresponding phases. Fig. 3.2 shows results of the RC-PTS for a QPSK modulated 64 sub-carrier OFDM system.

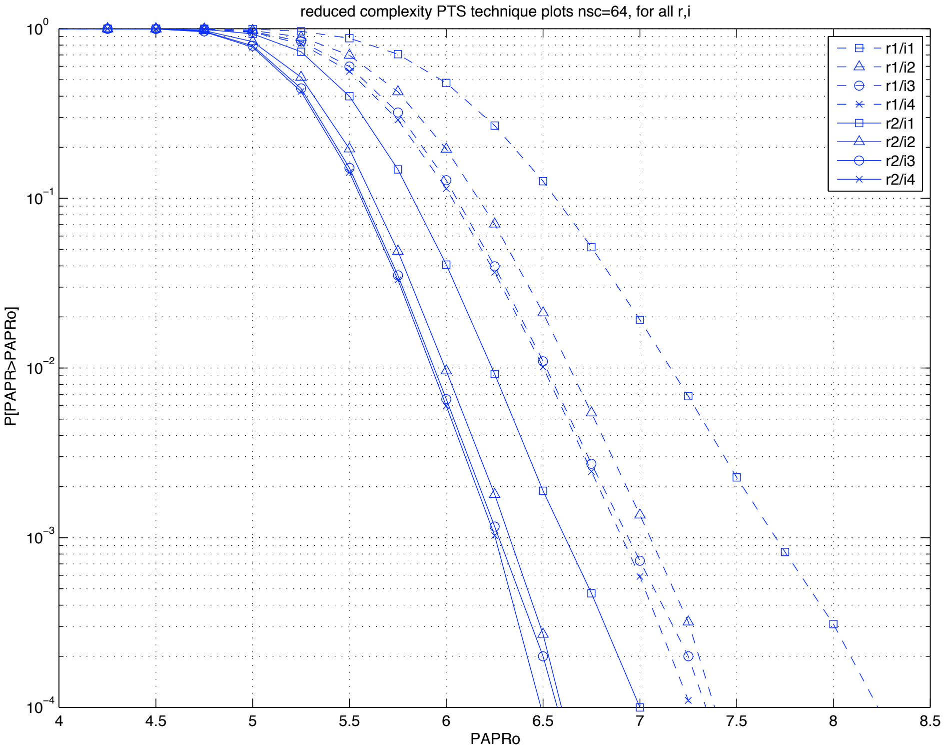

Figure 3.2: CCDFs of PAPR for reduced complexity PTS technique for $nsc = 64, L = 4; w = 4$ and $m = 8$.

The sub-carriers are over-sampled by a factor of $L = 4$ to interpolate the signal and estimate the true values of PAPR. The $r2/i3$ notation in the plot means that $r = 2$ is the Hamming distance radius used and $i = 3$ is the number of iterations in the RC-PTS. The radius **r** used in the implementation is either equal to 1 or 2 and the number of iterations **i** range from 1 to 4.

The CCDF (cumulative complementary distribution function) values are obtained from $P[PAPR > PAPRo]$. This means that the probability of the number of OFDM packets having PAPR values greater than that of the threshold value PAPRo is found. To estimate the correct values of CCDF, 100,000 OFDM packets are generated and the algorithm is implemented on these to obtain the PAPR values. A minimum of 2000 packets are expected to be simulated to obtain CCDF values close to about $0.1dB$ to the optimum results as in [10]. Fig. 3.3 shows the performance of a 128-subcarrier OFDM system.

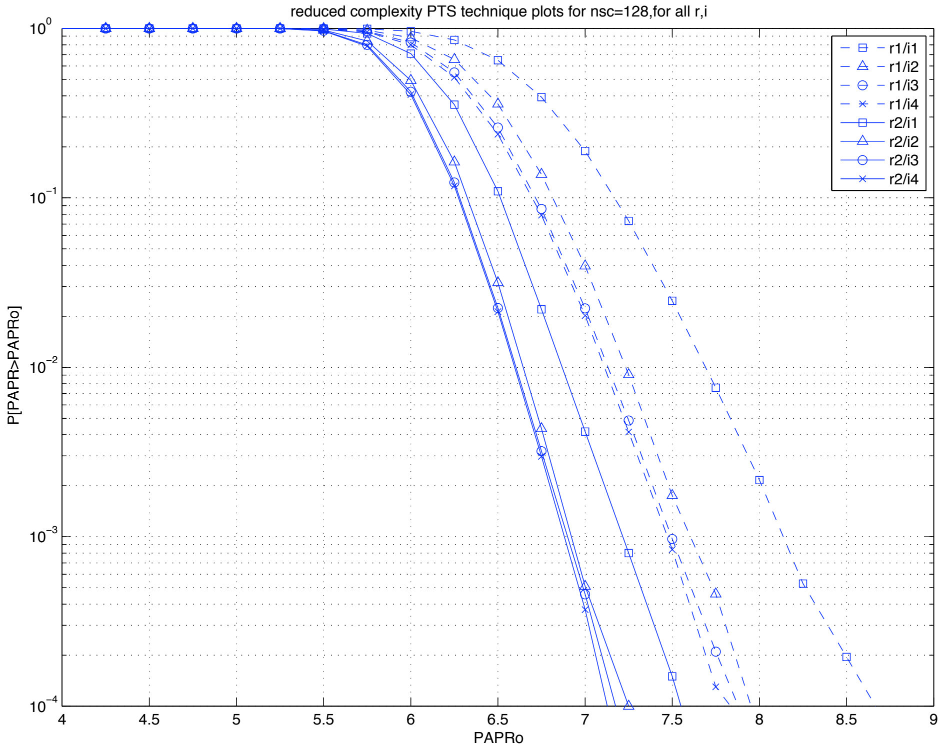

Figure 3.3: CCDFs of PAPR for reduced complexity PTS technique for $nsc = 128, L = 4; w = 4$ and $m = 8$.

The parameters $w = 4$ indicates the number of phases that are used in the algorithm. The phases in this case are $[+1, -1, +\jmath, -\jmath]$. The $m = 8$ indicates the number of partial transmit sequences used and finally $nsc = 128$ is the number of subcarriers in the OFDM system. Comparing Fig. 3.2 and Fig. 3.3 we can observe that PAPR values are high for higher number of subcarriers. The effect in the probability values of the RC-PTS algorithm is seen sooner to appear on the 64 subcarrier system starting from $4.75dB$. In the case of the 128 subcarrier system, the effect really starts after $5.5dB$ of threshold values.

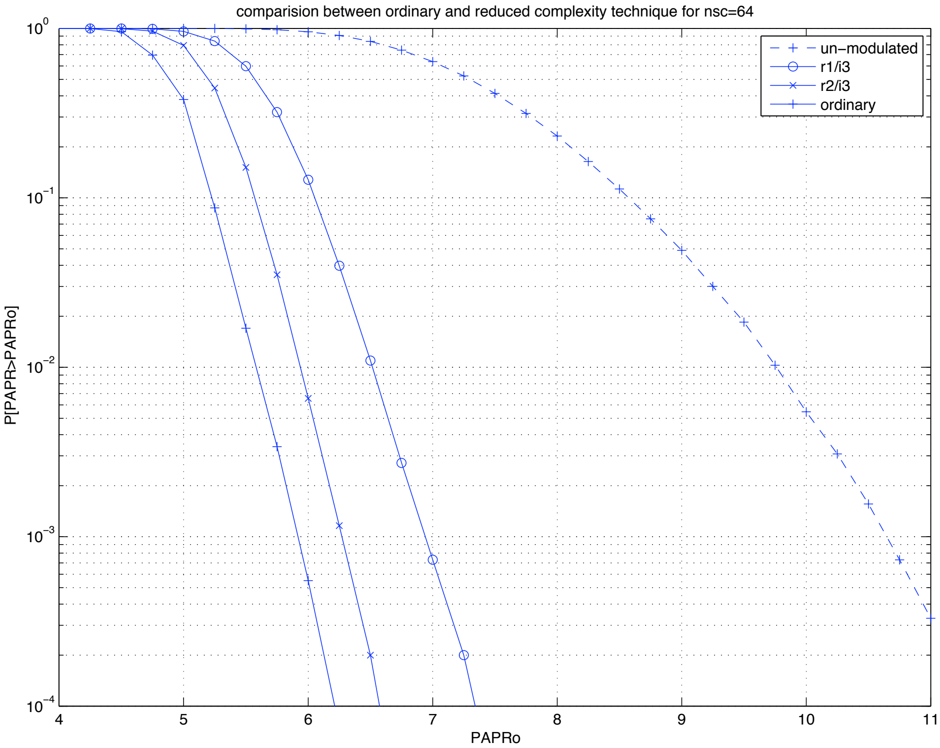

Figure 3.4: Performance analysis of reduced complexity and ordinary PTS technique for $nsc = 64, L = 4; w = 4$ and $m = 8$.

Fig. 3.4 shows a comparison between ordinary and the RC-PTS algorithms for a 64-subcarrier system. The unmodulated curve in the plot depicts the CCDF values when no PAPR reduction scheme is used. The ordinary curve depicts the ordinary PTS technique's performance in the PAPR reduction and the other curve depicts the RC-PTS performance. The impact of the Hamming radius on the PAPR reduction is seen by observing the $r1/i3$ and the $r2/i3$ curves of the plot. At $PAPRo = 6.5dB$, only 20 among the $100,000$ OFDM packets transmitted (used in this work) lie above the threshold level for $\mathbf{r} = 2$ which is very low when compared to 1096 packets when the Hamming distance $\mathbf{r} = 1$.

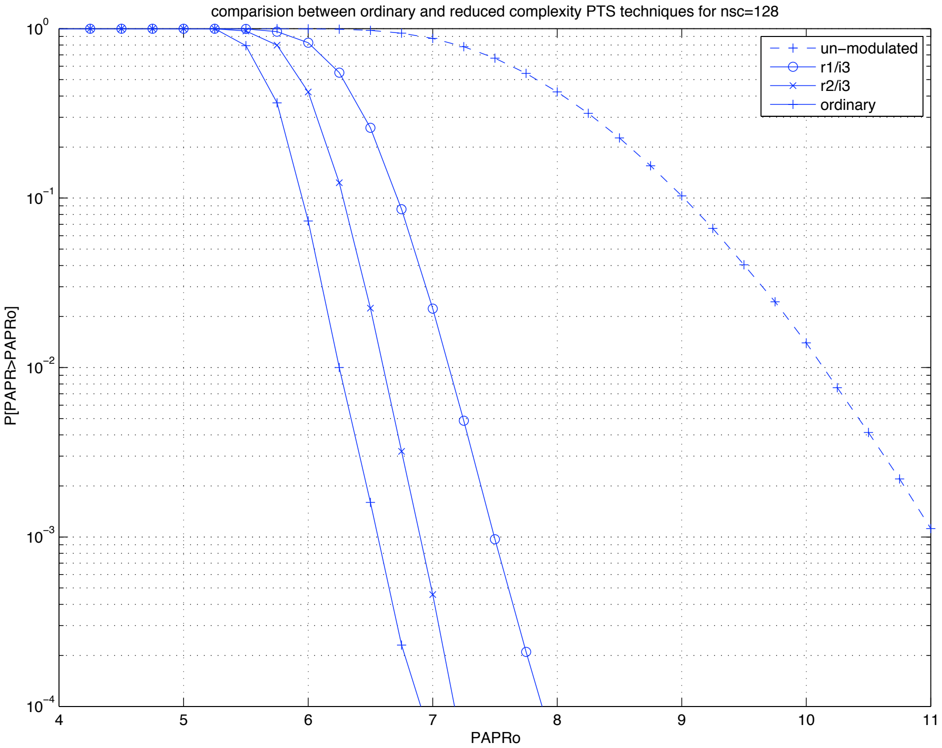

Figure 3.5: Performance analysis of reduced complexity and ordinary PTS technique for $nsc = 128, L = 4; w = 4$ and $m = 8$.

Fig. 3.5 shows performance comparison for the 128-subcarrier OFDM system. The behavior is similar to the case of $nsc = 64$. The efficiency of the RC-PTS curve can be analyzed when the $r2/i3$ curve and the unmodulated curve are compared. At $PAPRo = 7dB$, the unmodulated OFDM system has $87,519$ among the $100,000$ (used in this work) OFDM packets to have PAPR greater than the threshold value, which is very high compared to only 42 of these when RC-PTS is used with Hamming radius $\mathbf{r} = 2$.

3.4 COMPLEXITY REDUCTION

As discussed, the major complexity involved in the ordinary PTS technique is the search for the optimum phase vector generating the lowest PAPR. The RC-PTS improves the ordinary technique by reducing the number of phases that are explored. The search complexity of the RC-PTS algorithm is proportional to $\binom{M-1}{r} * w^r$. The variable M indicates the number of partial transmit sequences (M=8 in this report), w is the number of possible phases allowed for the search (w=4 in this report) and r is the radius used for the Hamming distance. So when the RC-PTS is implemented in the case of $\mathbf{r} = 1$, the complexity is proportional to $\binom{7}{1} * 4^1$ which is 28 and when $\mathbf{r} = 2$, this is proportional to $\binom{7}{2} * 4^2$ which is 336. These values for the number of phases searched is very low when compared to the number of phases searched in ordinary PTS technique which is $w^M = 4^8 = 65,536$. This decrease in complexity is huge when compared to the degradation in performance seen in the curves of Fig. 3.4 and Fig. 3.5.

Chapter 4

FURTHER STUDIES ON THE RC-PTS ALGORITHM

In this chapter, we perform two extensions to the RC-PTS algorithm given in [10]. First, the algorithm is studied using 256-subcarriers and 16-QAM constellations. Second, we propose and study a new idea of combining PTS technique with Clipping.

4.1 HIGHER MODULATION AND HIGHER SUBCARRIER SCHEMES

The idea of extending the RC-PTS algorithm to the higher modulation schemes is to achieve higher data transmission rates for which OFDM is generally known for. The 256-subcarrier system is expected to have larger PAPR values than the 64 and 128 subcarrier systems. The same parameters used in the RC-PTS algorithm are used for the extension of the work for the higher modulation schemes. The parameters are $L = 4, w = 4$ and $M = 8$.

Fig. 4.1 is the performance of the QPSK modulated 256-subcarrier OFDM system. The effect of the RC-PTS algorithm is not seen on the system until the PAPR values reach a threshold value of $6.25dB$ which is higher than the $4.75dB$ and $5.5dB$ observed in the QPSK modulated 64 and 128 subcarrier OFDM systems respectively. At $7.5dB$, the efficiency of the Hamming radius can be studied by observing that 3051 of the $100,000$ OFDM packets generated (in this work) have PAPR greater than the threshold for $\mathbf{r} = 1$ where as for $\mathbf{r} = 2$, only 42 of them have PAPR greater than the threshold.

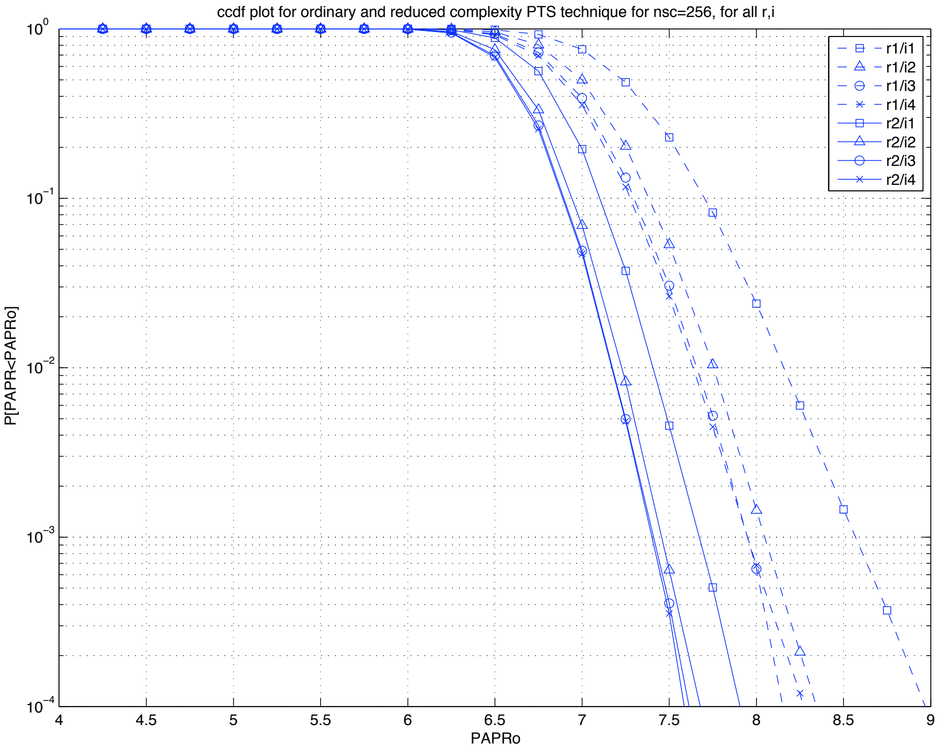

Figure 4.1: CCDFs of PAPR for reduced complexity PTS technique for $nsc = 256, L = 4; w = 4$ and $m = 8$.

The performance of the 16-QAM modulated OFDM system is shown in Fig. 4.2. The number of subcarriers used are $nsc = 64$ with other parameters remaining the same as before. Comparing this with the Fig. 3.2 we can say that the performance of the system is very close and same as the QPSK modulated 64-subcarrier system. This similarity in the performance for the 16-QAM and QPSK modulation schemes can be attributed to the idea of the number of different phases possible in the constellation of the systems. As the algorithm focuses on the modulation of the phases and attain the PAPR reduction, we expect the performance to be the same. So in the case of a selection of modulation schemes for better

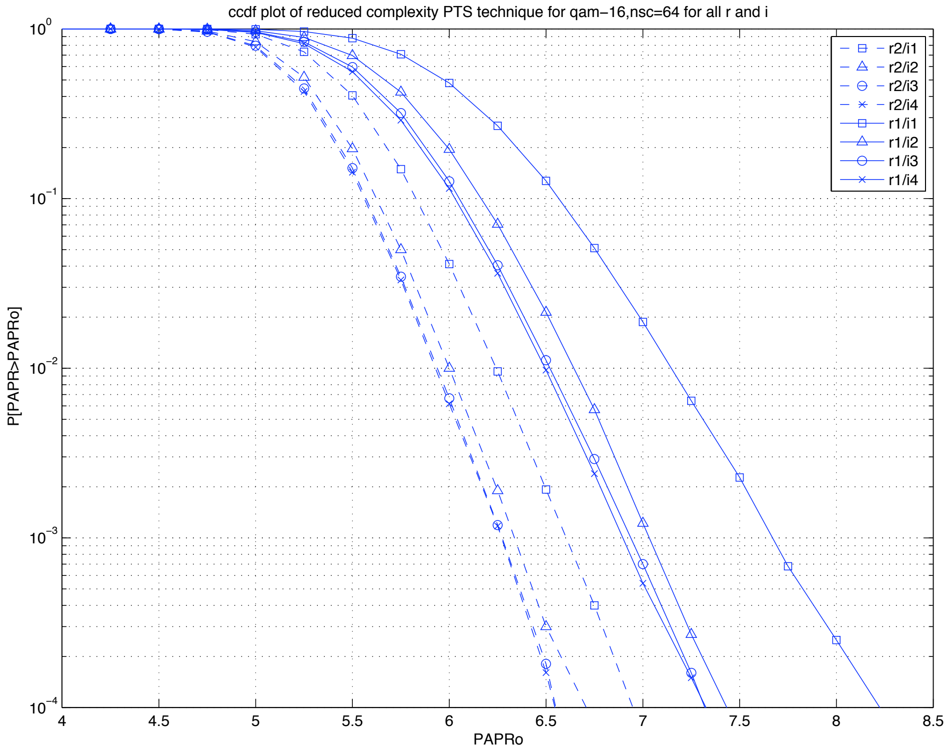

Figure 4.2: CCDFs of PAPR for reduced complexity PTS technique for 16-QAM,$nsc = 64, L = 4; w = 4$ and $m = 8$.

transmission rates, 16-QAM modulation can be preferred over QPSK when PAPR is considered as the selection criteria.

4.2 CLIPPING WITH PTS

Clipping the envelope of the signal to reduce the PAPR problem in OFDM systems is the easiest technique. PTS is an efficient PAPR reduction scheme without negative effect on the signal. The problem in PTS is complexity, which is not at all present in the clipping. So the idea of combining clipping and PTS is in a direction of reducing the complexity of PTS further than what is possible in ordinary RC-PTS and study the behavior of the system. The combined Clipping/PTS algorithm is as follows:

1. Calculate the PAPR value of the OFDM packet and compare with the threshold clipping level set.

2. If the packet has PAPR less than the level, forward the packet to the PTS or the RC-PTS stage for PAPR reduction.

3. If the condition in step 2 fails, then clip the power level of the signal to the clipping level and forward the packet without any further PAPR reduction.

4. Calculate the PAPR of all the OFDM packets generated and evaluate the probabilities for the CCDF plot by $P[PAPR > PAPRo]$.

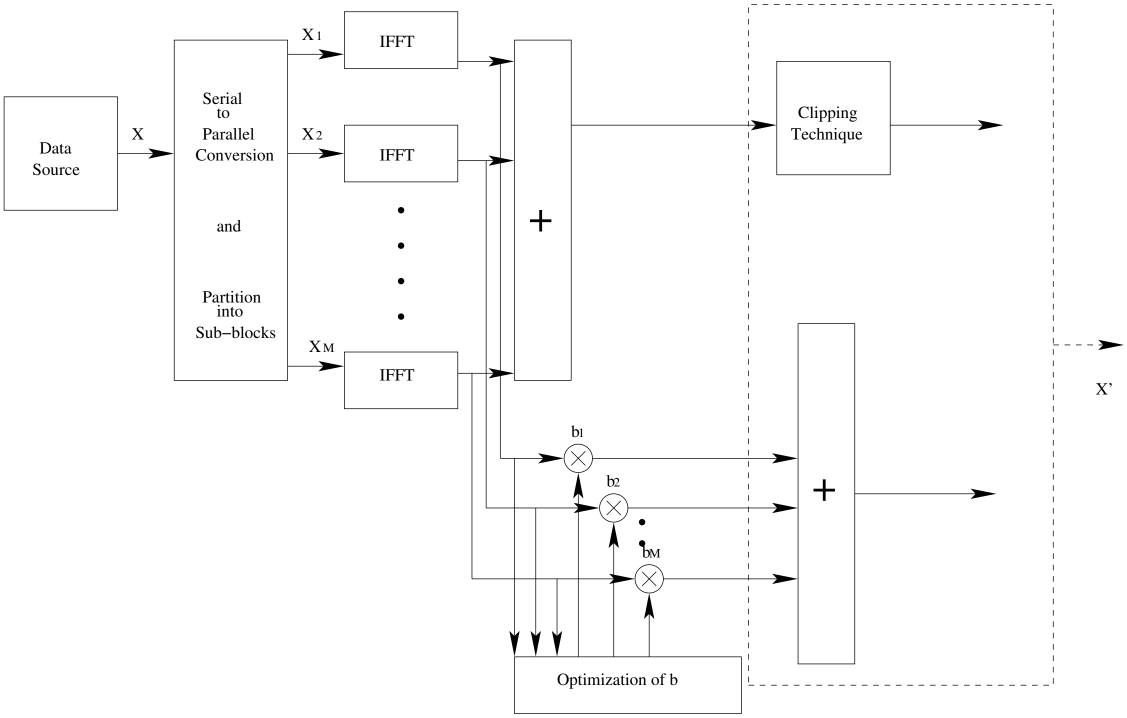

Figure 4.3: Block diagram representation of the proposed technique.

The block diagram representation of the proposed technique is given in Fig. 4.3. The OFDM symbols are checked for the PAPR values and those having greater than the threshold are clipped. The symbols having PAPR less than the clipping level are only passed through the PTS stage for phase vector optimization.

Fig. 4.4 is the behavior of the 64-subcarrier QPSK modulated OFDM system when clipping is combined with the ordinary PTS technique. The *level* $= 4$ in the figure indicates the clipping level is set at 4. The solid line on the right indicates the unmodulated system's curve without any PAPR reduction scheme involved and the dotted line to the extreme left without any design pertains to that of the implementation of the ordinary PTS technique. These two curves are highlighted to disaply a clear comparison in the performances.

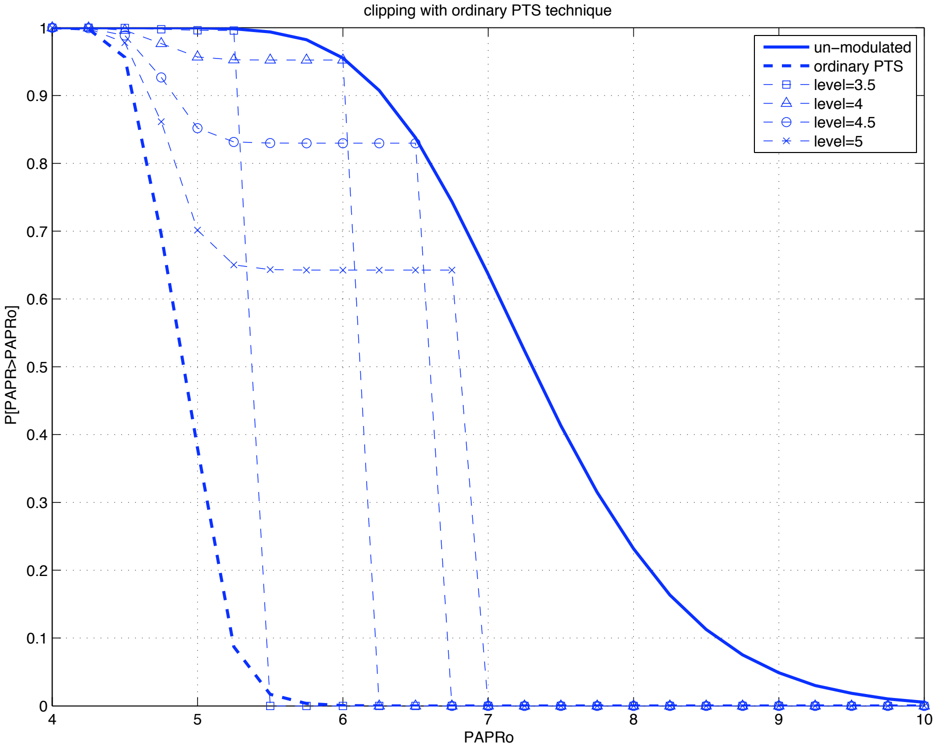

Figure 4.4: CCDFs of PAPR for clipping combined with ordinary PTS technique for $nsc = 64, L = 4; w = 4$ and $m = 8$.

The curves in between are obtained when clipping with PTS is used. The sudden fall in the curves can be related to the clipping level set as packets above those clipping levels are clipped and no further scheme is used for PAPR reduction.

For the third curve from the left, the clipping level set is $power = 4 = 6.021dB$, and this can clearly be seen from the curve starting to droop from $6dB$.

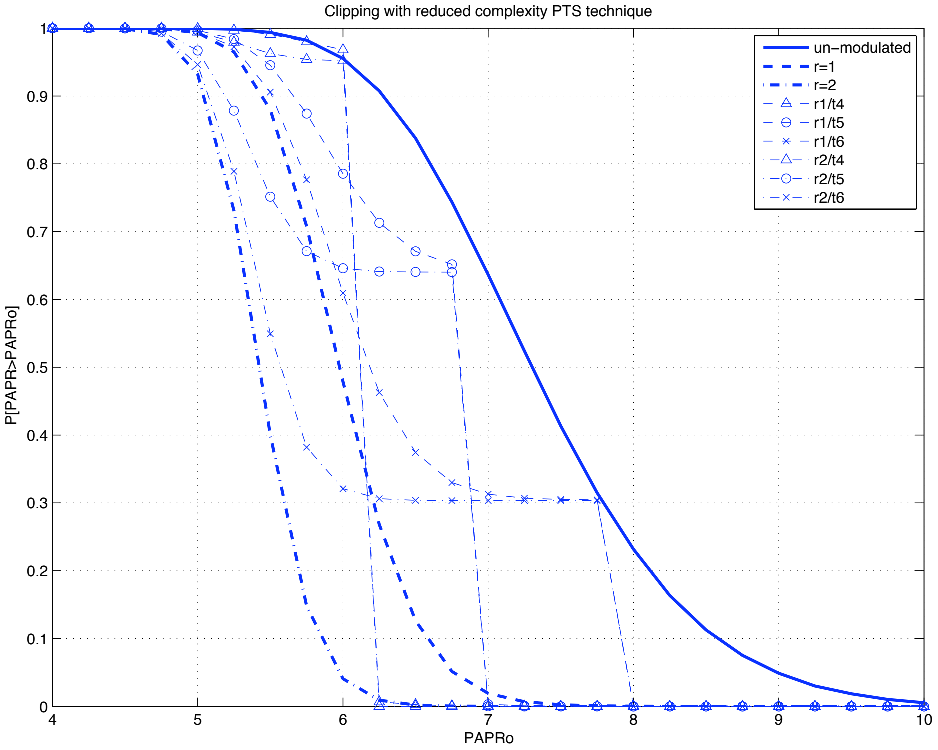

Figure 4.5: CCDFs of PAPR for clipping combined with reduced complexity PTS technique for $nsc = 64, L = 4; w = 4$ and $m = 8$.

Fig. 4.5 is the performance of the OFDM system when clipping is combined with the RC-PTS algorithm. The solid curve on the extreme right is again the unmodulated OFDM, the curve onto the left is when RC-PTS is used without clipping and a Hamming radius of $\mathbf{r} = 2$ is used. The bold dotted line in between is when the radius is $\mathbf{r} = 2$. The $r2/t5$ in the curve indicates the probability values when $\mathbf{r} = 2$ and the clipping level used is 5. For the same clipping level used, the RC-PTS with $\mathbf{r} = 2$ performs better than the $\mathbf{r} = 1$ scenario. The curves for different Hamming radius converge back to a point which can be considered to be the clipping level set. This can also be seen from the graph as, $level = 6 = 7.78dB$ and the corresponding curve starts drooping at a point close to $8dB$.

Fig. 4.6 is to show the performance of the proposed algorithm with both the ordinary and the RC-PTS techniques. The solid curves are used for the ordinary PTS technique and the dotted lines for the RC-PTS. The main idea of the graph is to show and prove that ordinary PTS technique performs better than the RC-PTS in any given conditions. The two clipping levels used are $4(6.02dB)$ and $5(6.99dB)$.

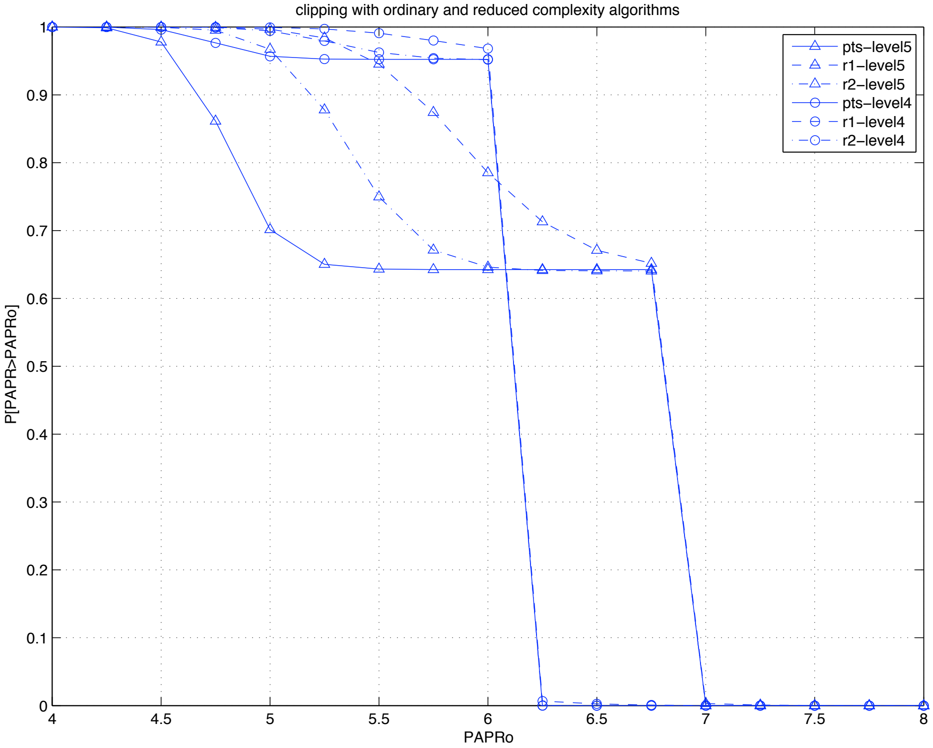

Figure 4.6: CCDFs of PAPR for clipping combined with ordinary and RC-PTS techniques for $nsc = 64$.

4.2.1 COMPLEXITY REDUCTION

The complexity reduction achieved by the proposed idea is being analyzed and the results are given in Fig. 4.7. The bar graph depicts the ratio of the OFDM packets the PTS technique operates on due to clipping. The ratio of the number of packets are obtained by using 100,000 OFDM packets and then implementing the proposed algorithm.The number of packets in the PTS stage are counted and the results are sketched.

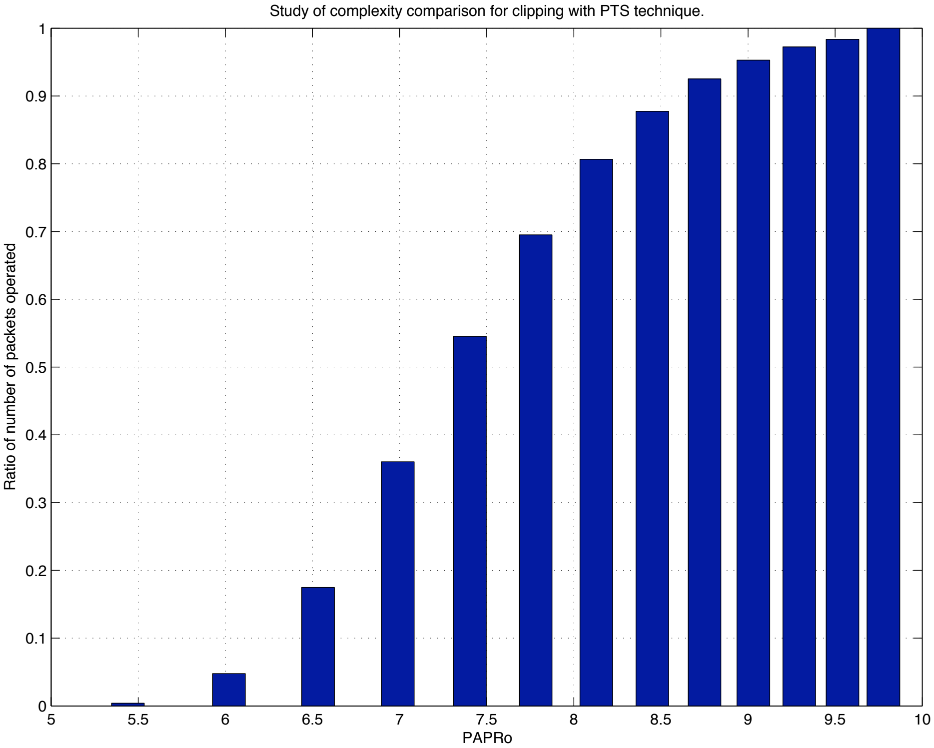

Figure 4.7: Complexity analysis for clipping combined with PTS techniques for $nsc = 64$.

It is observed that for low clipping levels, most of the OFDM packets get clipped and would not be present for the PTS stage. As the clipping level increases, less number of OFDM packets are clipped and more of them enter the PTS stage. Thus, the proposed algorithm provides a way to reduce complexity of PTS techniques. This reduction in complexity may adversely affect the BER performance of the system.

Chapter 5

CONCLUSIONS

5.1 CONCLUSIONS

In this report, PAPR reduction techniques for OFDM systems have been studied. Two types of reduction techniques, Clipping and Partial Transmit Sequences (PTS), are explored along with Reduced Complexity PTS (RC-PTS). The RC-PTS technique is extended to 256-subcarrier systems and 16-QAM modulations. A novel idea of combining Clipping with PTS and RC-PTS is proposed and the performance is evaluated. The performance of the RC-PTS is very close to the ordinary PTS technique with significant reduction in complexity.

Results show that for QPSK in the RC-PTS, the complexity can be reduced by about 64 times with a loss in performance of only $0.4dB$ when compared with the ordinary PTS technique. The extension of the RC-PTS to a 16-QAM system showed similar PAPR performance as that of a QPSK modulated OFDM system. The proposed idea of combining clipping with PTS and RC-PTS techniques can reduce the complexity of the PTS technique by a great extent at the cost of inducing non-linearity into the system.

5.2 FUTURE WORK

The complexity of the PTS technique for PAPR reduction of the OFDM systems is mainly based upon choosing the optimum phase rotations for partial transmit sequences. The phase selection is done from a very limited set. Extending the set of possible phase values can help to attain very promising results. Also more research is needed to perform PAPR reduction without transmitting additional information as is done in PTS to ensure proper BER performance.

Work also needs to be done to obtain proper synchronization in OFDM systems.

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